SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2015
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #2

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen.
- Board approved calculators may be used.
- Each Section is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 69

- Attempt questions 1 – 12.
- All questions are not of equal value.
- Unless otherwise directed give your answers in simplest exact form.

Examiner: *A.M.Gainford*

Section A (24 Marks)

Questions 1 to 7. (7 marks)

Indicate which of the answers A, B, C, or D is the correct answer. **Marks**
Write the answer on the separate answer sheet.

(1) The gradient of the tangent to the curve $y = (x-1)(x^2+1)$ at the point where $x = \frac{1}{2}$ is: **1**

A: $-\frac{4}{3}$

B: $\frac{3}{4}$

C: $\frac{4}{3}$

D: $-\frac{3}{4}$

(2) For what values of x is the curve $f(x) = 2x^2 - x^3$ increasing? **1**

A: $x > \frac{2}{3}$

B: $x < 0,\ x > \frac{4}{3}$

C: $x < \frac{2}{3}$

D: $0 < x < \frac{4}{3}$

(3) What is the slope of the line containing the points $(-9,2)$ and $(3,14)$. **1**

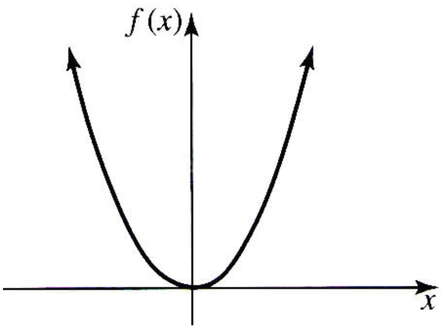
A: $\frac{4}{3}$

B: $-\frac{1}{2}$

C: 1

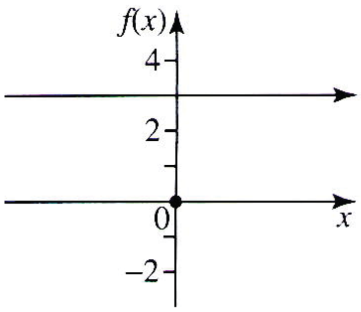
D: -2

(4) The figure below is the graph of $f\left(x\right)=3x^{2}$: **1**



Which of the following represents the graph of the second derivative?

A:



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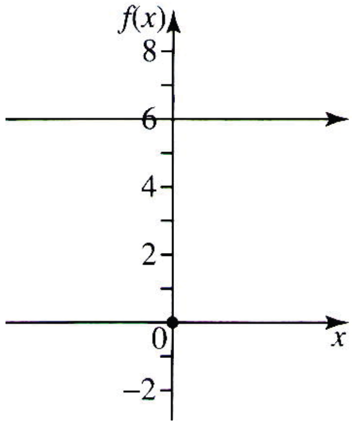

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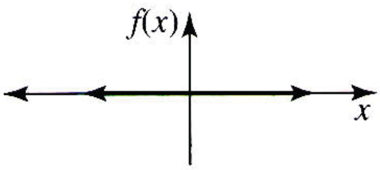

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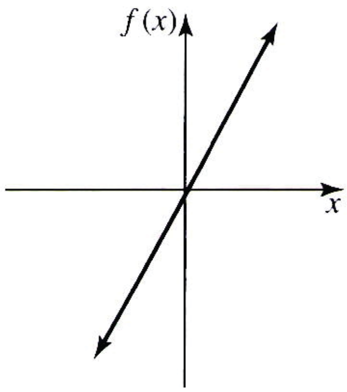


(5) Differentiate the following equation: $y = 6x^{-3}$ **1**

A: $\frac{dy}{dx} = -18x^{-2}$

B: $\frac{dy}{dx} = -12x^{-3}$

C: $\frac{dy}{dx} = -18x^{-4}$

D: $\frac{dy}{dx} = -3x^{-2}$

(6) It is known that $f''(a) = 0$. The point $(a, f(a))$ is:

A: a minimum turning point

B: a maximum turning point

C: a horizontal point of inflection

D: not determined (insufficient information)

(7) The graph with equation $y = x^2$ is translated 3 units down and 2 units to the right. Which equation represents the resulting graph?

A: $y = (x-2)^2 + 3$

B: $y = (x-2)^2 - 3$

C: $y = (x+2)^2 + 3$

D: $y = (x+2)^2 - 3$

Question 8 (17 marks) (Start a new booklet)

Marks

(a) Differentiate the following:

(i) $\ln x^3$ **1**

(ii) e^{2x+1} **1**

(iii) $e^x \ln x$ **1**

(b) Find

(i) $\int \left(3x^2 - 4x + 1\right) dx$ **2**

(ii) $\int \left(\frac{1}{x} - \frac{1}{x-1}\right) dx$ **2**

(iii) $\int \left(\frac{1}{e^{2x}}\right) dx$ **2**

(c) Evaluate

(i) $\int_{-1}^{3} (2x-1)\, dx$ **2**

(i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{\cos x}{1+\sin x}\right) dx$ **2**

(d) (i) Copy and complete the table for $f(x) = e^{x^2}$ correct to 4 decimal places. **2**

x	0	0.5	1	1.5	2
f(x)	1				

(ii) Use Simpson's Rule with the above 5 function values to find an approximation to $\int_0^2 e^{x^2}\, dx$ correct to 3 decimal places. **2**

Section B (23 Marks)

START A NEW BOOKLET

Question 9 (12 Marks)

Marks

(a) In an *x*-*y* plane in you answer booklet:

(i) Sketch the line through $C(-3,0)$ which makes an angle of 45° with the positive direction of the *x*-axis. Also sketch the line $x + y = 4$, which meets the first line at *A*, and the *x*-axis at *B*. **1**

(ii) Show that *AC* is perpendicular to *AB*. **1**

(iii) Find the equation of the line through *B*, which is parallel to *AC*. **2**

(iv) Show that the equation of the line through the point *C* parallel to the line *AB* is $x + y + 3 = 0$. **1**

(v) The lines from parts (iii) and (iv) meet at *D*. Find the coordinates of *D*. **2**

(vi) What type of quadrilateral is *ABDC*? Give reasons. **2**

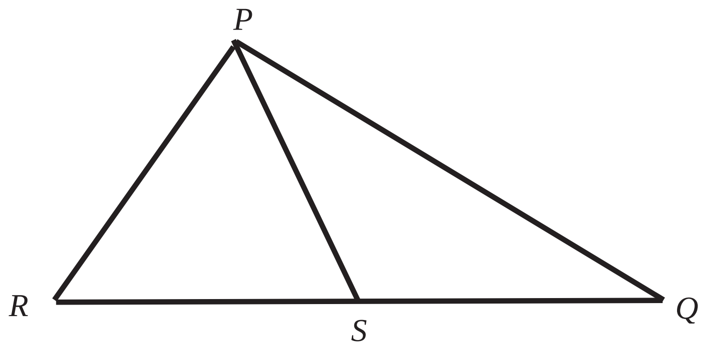
(b) A car's velocity *v* in metres per second is recorded each second as it accelerates along a drag strip. The table below gives the results. **3**

t (s)	0	1	2	3	4	5
$v\left(ms^{-1}\right)$	0	15	31	48	64	83

Given that the distance travelled may be found by calculating the area under a velocity/time graph, use the trapezoidal rule to estimate the distance travelled by the car in the first five seconds.

Question 10 (11 Marks)

(a)



In the diagram $\angle QPR = 90^0$, PS = SQ.

(i) Copy the diagram to your answer booklet.

(ii) Construct $ST \perp PQ$ (Sketch only). **1**

(ii) Prove that $PS = \frac{1}{2}RQ$. **3**

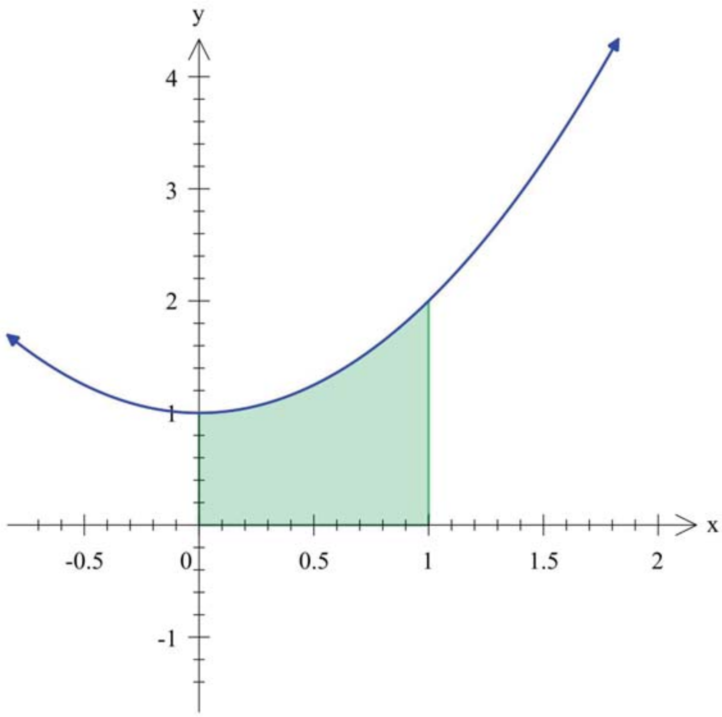
(b) A triangle has vertices *A*(4, 0), *B*(-4, 0) and *C*(0, 6).

(i) Draw a neat sketch of the triangle in your answer booklet. **1**

(ii) State the coordinates of *D* and *E*, the mid-points of *CA* and *CB* respectively. **1**

(iii) Show that the medians *BD* and *EA* meet on the *y*-axis. **3**

(c) **2**



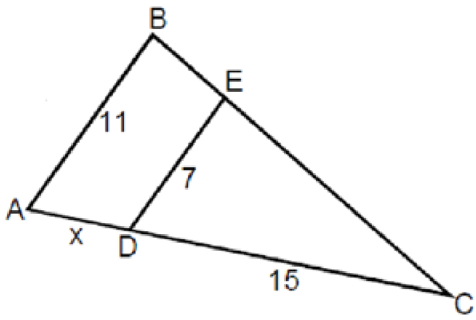
In the diagram the shaded region is bounded by the parabola $y = x^2 + 1$, the x-axis and the lines $x = 0$, and $x = 1$.
Find the volume of the solid formed when then shaded region is rotated about the x-axis.

Section C (22 Marks)

START A NEW BOOKLET

Question 11 (11 Marks)

(a) **3**



In the diagram above, $AB \parallel DE$.

Find the value of x, correct to 2 decimal places, giving full reasons.

(b) Consider the curve with equation $y = 2x^3 - 9x^2 + 12x - 3$.

(i) Find the co-ordinates of the stationary points and determine their nature. **4**

(ii) Find the co-ordinates of any points of inflexion. **2**

(iii) Sketch the curve for the domain $0 \le x \le 3$. (Do not attempt to find any x-intercepts.) **2**

Question 12 (11 Marks)

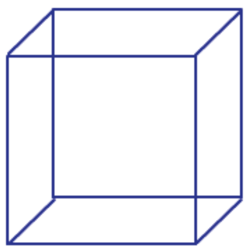
(a) For a certain curve $y = f(x)$, $f''(x) = 6x - 1$. **2**

Find the equation of the curve given that it passes through the point (1, -2) with gradient -3.

(b) (i) Differentiate $e^x\sqrt{x}$. **2**

(ii) Hence evaluate $\int_1^2 \frac{e^x(1+2x)}{\sqrt{x}}dx$. **2**

(c) A sealed tin rectangular box is to have a square base and a volume of 64 cm^3. If the length of the edge of the base is x cm:



(i) Express the height of the box in terms of x. **1**

(ii) Show that the total surface area y cm^2 of the box is given by **2**

$y = \frac{256}{x} + 2x^2$.

(iii) Find the minimum surface area of the box, and its dimensions. **2**

This is the end of the paper.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x,\ \ x > 0$

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2015

HSC Task #2

Mathematics

Suggested Solutions & Markers' Comments

QUESTION	Marker
1 – 7	–
8	VL see PSP
9 & 10	JC
11 & 12	RB

Multiple Choice Answers

1. B
2. D
3. C
4. B
5. C
6. D
7. B

Q.1 $y' = u'v + uv'$ where $u = x-1$

$= x^2+1 + (x-1) \times 2x$ $\quad u' = 1$

$= x^2+1 + 2x^2 - 2x$ $\quad v = x^2+1$

$= 3x^2 - 2x + 1$ $\quad v' = 2x$

$$f'\left(\tfrac{1}{2}\right) = 3\left(\tfrac{1}{2}\right)^2 - 2\left(\tfrac{1}{2}\right) + 1$$

$$= \frac{3}{4} - 1 + 1$$

$$= \frac{3}{4}$$

$\boxed{B}$

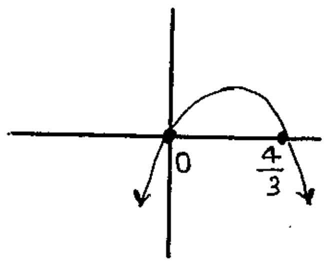
Q.2 $f'(x) = 4x - 3x^2$

curve increases when $f'(x) > 0$

$$4x - 3x^2 > 0$$

$$x(4-3x) > 0$$

$$\therefore \quad 0 < x < \frac{4}{3}$$



$\boxed{D}$

Q.3 slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{14-2}{3-(-9)}$$

$$= 1$$

$\boxed{C}$

Q. 4

$f(x) = 3x^2$

$f'(x) = 6x$

$f''(x) = 6$

$\boxed{B}$

Q. 5

$y = 6x^{-3}$

$$\frac{dy}{dx} = -18x^{-4}$$

$\boxed{C}$

Q. 6

$\boxed{D}$

Q. 7

$y = (x-2)^2 - 3$

translated 3 units down

translated 2 units to the right

$\boxed{B}$

Question 8

(a) (i) $\frac{d}{dx} \ln x^3 = \frac{d}{dx} 3 \ln x$

$$= \frac{3}{x}$$

a)(i) Those who wrote the following received no marks because they are all incorrect:

$(3x^2)^{-1}$, $\frac{1}{x^3}$, $\frac{x}{3}$, $\frac{3 \ln x^2}{\ln x^3}$, $\frac{2x}{x^3}$, $3 \ln x^2$, $3x^3$, $3e^x$, $\frac{3}{x^2}$, $3 \ln e^x$, $3x^2 \ln x^3$

Those who wrote $\frac{3x^2}{x^3}$ received $\frac{1}{2}$ out of 1 mark because they should have simplified their answer to $\frac{3}{x}$.

(ii) $\frac{d}{dx} e^{2x+1} = 2e^{2x+1}$

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a)(ii) Those who wrote the following received no marks because they were all incorrect:

$2e^{2x}$, e^2, e^{2x+1}, $2xe^{2x+1}$, $(2x+1)e^{2x+1}$, $(2x+1)e^{2x}$, $\frac{1}{2}e^{2x+1}$

(iii) $\frac{d}{dx} e^x \ln x = u'v + uv'$ where $u = e^x$

$= e^x \ln x + \frac{e^x}{x}$ $\quad u' = e^x$

$\quad v = \ln x$

$= e^x\left(\ln x + \frac{1}{x}\right)$ $\quad v' = \frac{1}{x}$

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a)(iii) The product rule should be used.
Those who did not use the product rule wrote the following which were all incorrect:

$e^x \times \frac{1}{x} = \frac{e^x}{x}$, $e^x \ln x$, $\frac{1}{x^x}$, x^{-e^x}, e^x, $2xe^x \ln x$

(b) (i) $\int (3x^2 - 4x + 1)\,dx = x^3 - 2x^2 + x + C$

(ii) $\int \left(\frac{1}{x} - \frac{1}{x-1}\right) dx = \ln x - \ln(x-1) + C$

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b) → 1 mark was lost if the constant was not included.
b)(i) well answered overall
b)(ii) The following scored zero marks:

$-1-(x-1)^{-1}+c$, $\ln x + \ln(x-1)+c$, $1+c$, $-x+c$,

$\frac{-2}{x-1}$, $\frac{x^0}{0}$ = undefined, $\frac{\ln(x)}{\ln(x-1)}$, $\frac{1}{x}+C$, $\ln x - \ln x - \ln x$

(iii) $\int \left(\frac{1}{e^{2x}}\right) dx = \int e^{-2x}\,dx$

$= \frac{e^{-2x}}{-2} + c$

(b) (iii) If the negative sign was omitted from the first term, one mark was lost

i.e. $\dfrac{e^{-2x}}{-2} + c$ is the correct answer $\left(\text{not } \dfrac{e^{-2x}}{2} + c\right)$

This question was not answered correctly overall with incorrect answers such as:

$\dfrac{\ln e^{2x}}{2e^{2x}}$, $\ln e^{2x} + c$, $e^{-2x} + c$, $2x + c$, $\dfrac{e^{2x-1}}{2x}$,

$\dfrac{x}{e^{2x}} + c$, $\dfrac{1}{e^{x}} + c$, $\dfrac{1}{2}\ln(e^{2x})$, $-3e^{-3x} + c$,

$\dfrac{e^{-2x+1}}{-2x+1} + c$, $\ln e^{2x} \cdot \dfrac{1}{2e^{2x}}$, $\dfrac{1}{e^{2x^2}} + c$

(c) (i) $\displaystyle\int_{-1}^{3} (2x-1)\,dx = \left[x^2 - x\right]_{-1}^{3}$

$$= (3^2 - 3) - \left((-1)^2 - (-1)\right)$$

$$= (9-3) - (1+1)$$

$$= 4$$

(c) (i) $\left[x^2 - x\right]_{-1}^{3}$

$= (3^2 - 3) - ((-1)^2 - (-1))$

$= (9-3) - (1+1)$

$= 4$

If grouping symbols were not used, students incorrectly wrote $6-(-2)$
$= 8$

or incorrectly wrote
$6 - 1 + 1$
$= 6$

or incorrectly wrote
$6 - (1-1)$
$= 6$

The negative sign on the lower limit was incorrectly left out (1 was used instead of -1)

Most students answered this correctly because grouping symbols were used.

(c) (ii) $\displaystyle\int_{\pi/6}^{\pi/3} \left(\frac{\cos x}{1 + \sin x}\right) dx = \left[\ln(1 + \sin x)\right]_{\frac{\pi}{6}}^{\pi/3}$

$= \ln\left(1 + \frac{\sqrt{3}}{2}\right) - \ln\left(1 + \frac{1}{2}\right)$

$= \ln\left(\frac{2+\sqrt{3}}{2}\right) - \ln\left(\frac{3}{2}\right)$

$= \ln\left(\dfrac{\frac{2+\sqrt{3}}{2}}{\frac{3}{2}}\right)$

$= \ln\left(\frac{2+\sqrt{3}}{3}\right)$

(d) (i)

x	0	0.5	1	1.5	2
$f(x)$	1	1.2840	2.7183	9.4877	54.5982

(ii)
$$\int_0^2 e^{x^2}\,dx \approx \frac{h}{3}\left[f(0)+f(2)+4\big(f(0.5)+f(1.5)\big) + 2\big(f(1)\big)\right]$$

$$= \frac{\frac{2-0}{4}}{3}\left[1+54.5982+4(1.2840+9.4877) + 2(2.7183)\right]$$

$$= 17.354 \text{ (correct to 3 decimal places)}$$

(d) (i) Some students did not round to 4 decimal places (instead they rounded to 3 decimal places and lost 1 mark).

When completing the table, approximations were not written and instead exact values $e^{\frac{1}{4}}$, e^1, $e^{2\frac{1}{4}}$, e^4 were written. No marks were awarded for exact values.

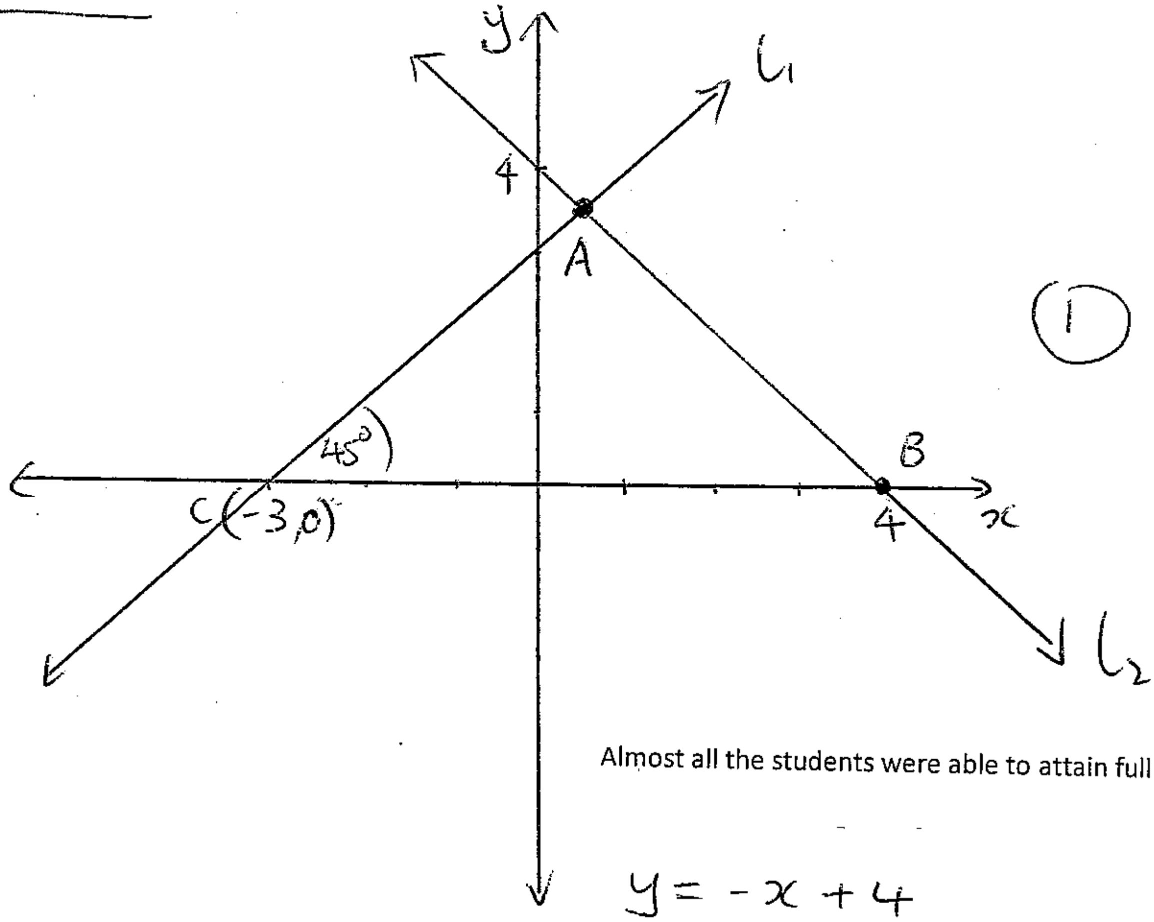
$\frac{1}{2}$ mark was lost for each incorrect approximation.

(d)(ii) Overall, Simpson's Rule was written and applied correctly. In a few cases, $\frac{h}{3}$ was incorrectly calculated as $\frac{2-0}{4} = \frac{1}{2}$ (Instead of $\frac{1}{6}$)
$\frac{1}{2}$ mark was lost if $\frac{h}{3}$ was incorrect.

If Simpson's Rule was written and applied correctly however the final answer was not rounded to 3 decimal places or incorrect, $\frac{1}{2}$ mark was lost.

Question 9

a) (i)



Almost all the students were able to attain full marks

(ii) $m_{l_1} = \tan 45^\circ$

$m_{l_1} = 1$

$y = -x + 4$

$m_{l_2} = -1$

$m_{l_1} \times m_{l_2} = 1 \times -1 = -1$ 1

$\therefore AC \perp AB$.

Almost all the students were able to attain full marks

(iii) $y - 0 = 1(x - 4)$ 1

$\boxed{y = x - 4}$ 1

Almost all the students were able to attain full marks

(iv) $y - 0 = -1(x + 3)$

$y = -x - 3$

$x + y + 3 = 0$ 1

Almost all the students were able to attain full marks

Question 9 (continued)

a) (v) $y = x - 4$ —— (1)

$x + y + 3 = 0$ —— (2)

Sub (1) in (2),

$x + x - 4 + 3 = 0$

$2x - 1 = 0$

$\boxed{x = \frac{1}{2}}$ (1)

$y = \frac{1}{2} - 4$

$\boxed{y = -3\frac{1}{2}}$ (1)

$\boxed{\therefore D = \left(\frac{1}{2}, -3\frac{1}{2}\right)}$

Almost all the students were able to attain full marks

(vi) ABCD is a square (1)

* All sides equal, adjacent sides are perpendicular. (1)

This part was poorly done, most students weren't able to identify the type of quadrilateral

Question 9 (continued) (3)

b) $d = \frac{h}{2}\left[f(a) + 2f(x_n) + f(b)\right]$ (1)

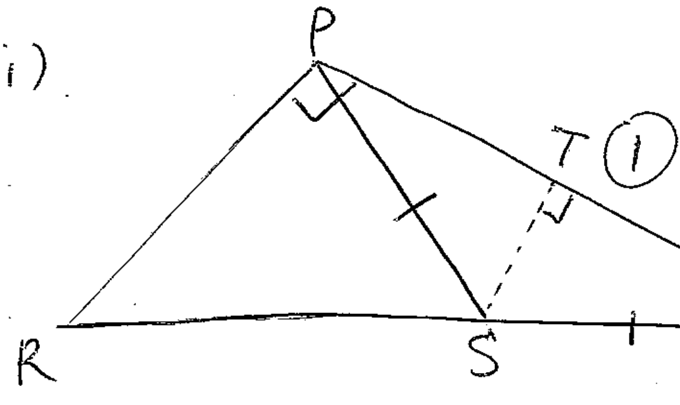
$= \frac{1}{2}\left[0 + 2(15+31+48+64) + 83\right]$ (1)

$\boxed{d = 199\frac{1}{2}\ m}$ (1)

Most students were able to attain full marks

Question 10:

a) (i)



Almost all the students were able to attain full marks

(iii) Let $\angle SPT = x$

$\angle SQT = \angle SPT = x$ (base $\angle$s of isos. $\triangle$)

$\angle RPS = 90^\circ - x$ (comp. $\angle$)

$\angle PRQ = 180^\circ - 90^\circ - x$ ($\angle$s sum of a right-$\angle$ $\triangle$) (1)

$= 90^\circ - x$

$\therefore \angle PRQ = \angle RPS = 90^\circ - x$

$\therefore \triangle RSP$ is isosceles with $PS = RS$ (1)

$\therefore PS = RS = SQ$

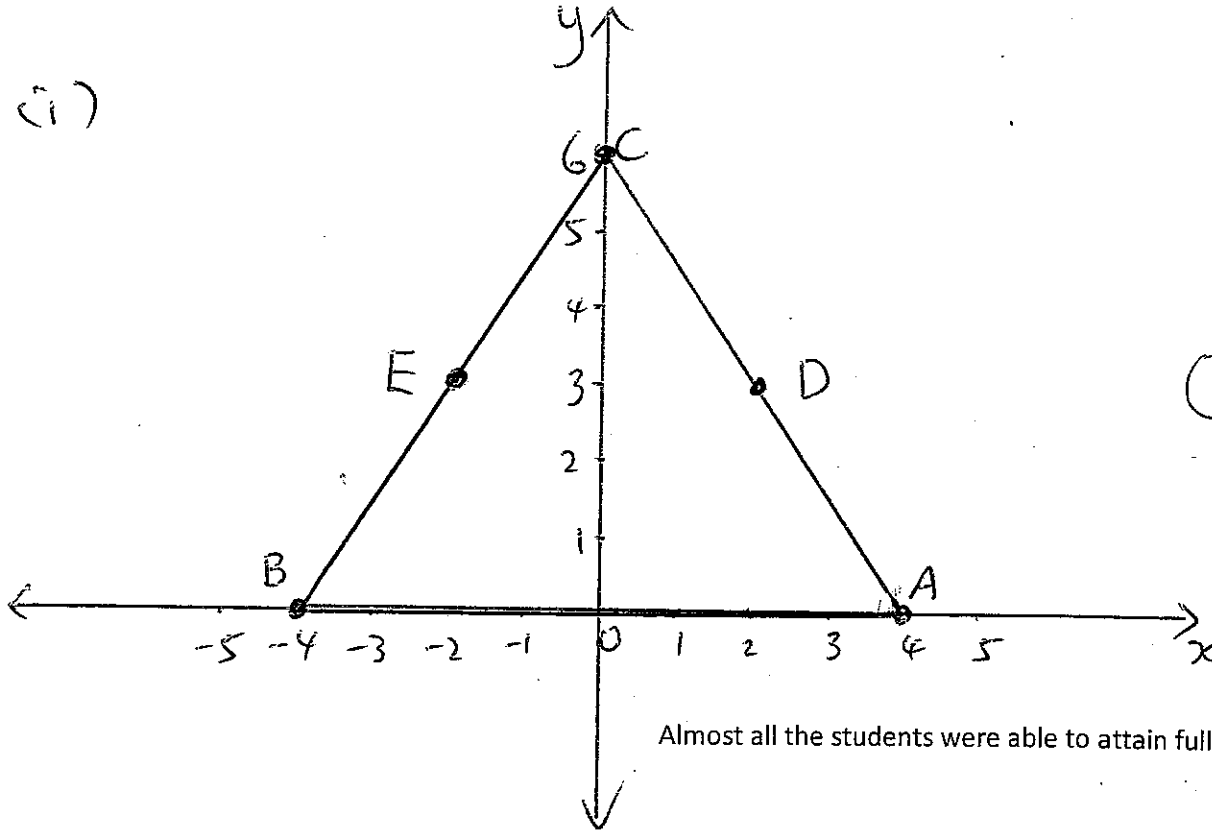
$\therefore PS = \frac{1}{2}(RS + SQ)$ (1)

$PS = \frac{1}{2} RQ$ //

Most students were able to write out a logical proof for this question

Question 10 (continued)

b) (i)



1

Almost all the students were able to attain full marks

(ii) D(2, 3) $\frac{1}{2}$

E(−2, 3) $\frac{1}{2}$

Almost all the students were able to attain full marks

(iii) $m_{BD} = \frac{3-0}{2+4}$ $\qquad$ $m_{AE} = \frac{3-0}{-2-4}$ $\qquad$ 1

$m_{BD} = \frac{1}{2}$ $\qquad$ $= -\frac{1}{2}$

Equation of BD: $y - 0 = \frac{1}{2}(x+4)$

$y = \frac{1}{2}x + 2$ —— (1)

Equation of AE: $y - 0 = -\frac{1}{2}(x-4)$ $\qquad$ 1

$y = -\frac{1}{2}x + 2$ —— (2)

Equate (1) & (2)

$\frac{1}{2}x + 2 = -\frac{1}{2}x + 2$ $\qquad$ $y = \frac{1}{2}(0) + 2$

$\boxed{x = 0}$ $\qquad$ $\boxed{y = 2}$ $\qquad$ 1

∴ Medians meet at (0, 2) which is on the y-axis.

Most students did not remember the meaning of 'median' and less than half were able to attain full marks in this part

Question 10 (continued)

c) $V = \pi \int_a^b y^2 \, dx$

$= \pi \int_0^1 (x^2+1)^2 \, dx$

$= \pi \int_0^1 x^4 + 2x^2 + 1 \, dx$ 1

$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_0^1$

$= \pi \left[\frac{1}{5} + \frac{2}{3} + 1 - 0 \right]$

$\boxed{\therefore V = \frac{28\pi}{15}}$ 1

Deduct $\frac{1}{2}$ for not including π.

Almost all the students were able to attain full marks

Assessment Task 2 Unit 2015
March/April Section C

11

11

a)

In $\triangle ABC$ and $\triangle DEC$,

$\hat{C}$ is common

$B\hat{A}C = E\hat{D}C$ angles in the corresponding position as $AB \parallel DE$ given

1

$\therefore \triangle ABC \parallel\!| \triangle DEC$ 2 angle test

or stating the proportional division theorem
"A line parallel to 1 side of a triangle divides the other sides in the same ratio"

thus $\dfrac{AB}{DE} = \dfrac{AC}{DC} = \dfrac{BC}{EC}$ 1

$\dfrac{11}{7} = \dfrac{x+15}{15}$

$11 \times 15 = 7(x+15)$

$165 = 7x + 105$

$7x = 60$

$x = \dfrac{60}{7} \doteqdot 8.57$ 2DP 1

* Well answered but students did get the ratio of sides wrong.

11 (b) $y = 2x^3 - 9x^2 + 12x - 3$

$y' = 6x^2 - 18x + 12$

$y'' = 12x - 18$

Well answered but max/min has to be established. Sign change for inflexion established. Graph domain $0 \le x \le 3$ only and y values then.

(i) Stat points exist when $y' = 0$

$6x^2 - 18x + 12 = 0$

$\div 6$ $\quad x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 1, \quad x = 2$ ①

at $x=1$, $y = 2 - 9 + 12 - 3 = 2$ $\quad (1, 2)$ ①

at $x=2$, $y = 16 - 36 + 24 - 3 = 1$ $\quad (2, 1)$

at $(1,2)$ $\quad y'' = 12 - 18 = -6 < 0$ MAX st pt. ①

at $(2,1)$ $\quad y'' = 24 - 18 = 6 > 0$ min st pt. ①

(ii) Inflexions occur when $y'' = 0$ and there is a sign change.

$y'' = 12x - 18 = 0$

$12x = 18$

$x = 1\frac{1}{2}$ ①

at $x = 1\frac{1}{2}$ $\quad y = 6.75 - 20.25 + 18 - 3 = 1\frac{1}{2}$ $\quad (1\frac{1}{2}, 1\frac{1}{2})$

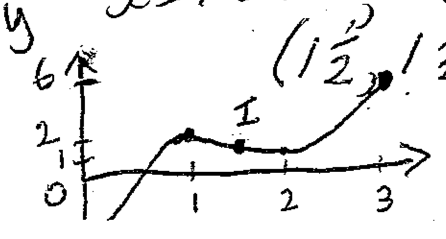
Sign change?

at $x = 1.5 - \varepsilon$, (1.4) $\quad y'' = 12 \times 1.4 - 18 = -1.2 < 0$ ①

$x = 1.5 + \varepsilon$ $\quad (1.6)$, $\quad y'' = 12 \times 1.6 - 18 = 1.2 > 0$

$(1\frac{1}{2}, 1\frac{1}{2})$ is a pt of inflexion

(iii)



②

(12) (a) $f''(x) = 6x - 1$

$f'(x) = \int (6x-1)\, dx$

$f'(x) = \frac{6x^2}{2} - x + C_1$

Data

$x=1$, $f'(x) = -3$ $\quad -3 = 3 - 1 + C_1$

$C_1 = -5$ (1)

$f'(x) = 3x^2 - x - 5$

$f(x) = \int (3x^2 - x - 5)\, dx$

$= \frac{3x^3}{3} - \frac{x^2}{2} - 5x + C_2$

$f(x) = x^3 - \frac{x^2}{2} - 5x + C_2$

data $(1,-2)$ $\quad -2 = 1 - \frac{1}{2} - 5 + C_2$

$C_2 = 2\frac{1}{2}$

$\therefore f(x) = x^3 - \frac{x^2}{2} - 5x + 2\frac{1}{2}$. (1)

/11

Often made mistakes.

; found $f'(x)$ but not the first constant.

; many found $f(x)$ but not the 2nd constant

; many students left out the question data completely

12 (b) (i) $\frac{d}{dx}\left(e^{x}.x^{\frac{1}{2}}\right)$

$= e^{x} \times \frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}.e^{x}$

$= \frac{e^{x}}{2\sqrt{x}} + e^{x}.\sqrt{x} = \frac{e^{x}+e^{x}.2x}{2\sqrt{x}}$

Most students handled the product rule well. Many though did not simplify enough to help in part (ii)

$= \frac{e^{x}(1+2x)}{2\sqrt{x}}$ not rationalised. (far enough) 2

(ii) $2\int_{1}^{2} \frac{e^{x}(1+2x)}{2\sqrt{x}}\,dx = \left[2e^{x}\sqrt{x}\right]_{1}^{2}$

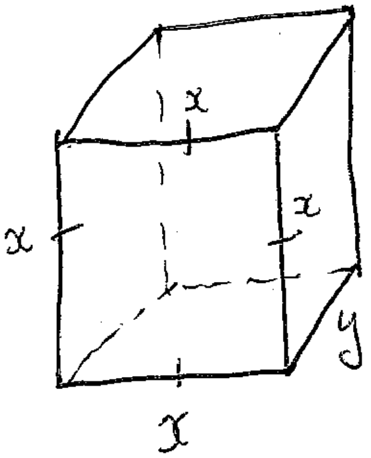
$= 2\left(e^{2}.\sqrt{2} - e^{1}.\sqrt{1}\right)$

$= 2\left(\sqrt{2}e^{2} - e^{1}\right)$

$= 2e\left(\sqrt{2}e - 1\right)$ exact. 2

Generally well answered but the $2\int$ was often missing.

12 (c)



(i) $V = x \times x \times y$ $\quad$ y = height

$64 = x^2 y$

$y = \frac{64}{x^2}$ $\quad$ height $y = \frac{64}{x^2}$ $\quad$ ① easily found

(ii) total S.A $= x^2 + x^2 + xy + xy + xy + xy$

$= 2x^2 + 4xy$

$= 2x^2 + 4x \times \frac{64}{x^2}$

$y = \frac{256}{x} + 2x^2$ $\quad$ ② well answered.

(iii) $y = 256x^{-1} + 2x^2$

$y' = -256x^{-2} + 4x$

$y'' = 512x^{-3} + 4$

When $y' = 0$ $\quad$ $-\frac{256}{x^2} + 4x = 0$

obviously $x \neq 0$ $\quad$ $\frac{-256 + 4x^3}{x^2} = 0$

$4x^3 = 256$

$x^3 = 64$

$x = 4$ cm

If dimensions are $4 \times 4 \times 4$

$$\frac{64}{4^2}$$

then $y'' = \frac{512}{4^3} + 4$

$= 8 + 4 = 12$

$\Rightarrow$ min value

So minimum S.A is when dimensions are $4 \times 4 \times 4$ cm and its value

SA is $2 \times 4^2 + \frac{256}{4}$

$= 32 + 64$

$= 96\ cm^2$

2

a minimum was often not found by either using y' or y''.
Dimensions often not finally quoted
The minimum surface area often not found as a conclusion.